Algebraic Geometry II
Homework 1
Due Friday, January 23

(1) Let $\pi : X \to Y$ be a continuous map, and $\mathcal{F}$ a sheaf on X. Show that $\pi_*\mathcal{F}$ is a sheaf on Y.

(2) Let $\mathcal{F}$ and $\mathcal{G}$ be sheaves on X, and $f_1,\ f_2 : \mathcal{F} \to \mathcal{G}$ maps of sheaves. Show that if f_1 and f_2 induce the same map on stalks, then $f_1 = f_2$.

(3) Let $\mathcal{F}$ be a presheaf. Show that $\mathcal{F}^{sh}$, the sheafification of $\mathcal{F}$, is a sheaf, and there is a map $\mathcal{F} \to \mathcal{F}^{sh}$ which induces an isomorphism on stalks.

(4) Let $p \in X$ and $\mathcal{F}$ a sheaf on X. Show that $\mathcal{F}_p \cong \operatorname{inj\,lim}_{p \in U} \mathcal{F}(U)$.

(5) Let $Spec(R)$ be an affine scheme. Show that the following topologies are the same. The first has as closed sets those of the form $\{\mathfrak{p} : \mathfrak{p} \supset I\}$ for each ideal I. The second has as a basis for the open sets $D(f) = \{\mathfrak{p} : f \notin \mathfrak{p}\}$.

(6) Show that given a space X, an open cover of X by sets U_α, sheaves $\mathcal{F}_\alpha$ on U_α, and isomorphisms $\varphi_{\alpha\beta} : \mathcal{F}_\alpha|_{U_\alpha \cap U_\beta} \to \mathcal{F}_\beta|_{U_\alpha \cap U_\beta}$ such that for any α, β, and γ, $\varphi_{\beta\gamma} \circ \varphi_{\alpha\beta} = \varphi_{\alpha\gamma}$ on $U_\alpha \cap U_\beta \cap U_\gamma$, there is a unique sheaf $\mathcal{F}$ such that $\mathcal{F}|_{U_\alpha} = \mathcal{F}_\alpha$.

(7) Show that on a Noetherian topological space, any closed set can be written uniquely as a finite union of irreducible sets.

(8) Describe $Spec(\mathbb{R}[x])$.

(9) Let $f \in \mathbb{C}[x]$. Suppose its image in $\mathbb{C}[x]/(x-1)^2$ is $2 - \pi(x-1)$. What the derivative of f at 1?