Algebraic Geometry II
Homework 10
Due Friday, April 17

(1) Let C_1 and C_2 be curves and $f : C_1 \to C_2$ a finite map of curves over a field of characteristic 0. Show that Ω_{C_1/C_2} is a torsion sheaf, and its scheme-theoretic support is the ramification divisor of f.

(2) Prove the Riemann-Hurwitz formula.

(3) Show that if $\pi : X \to Y$ is an affine morphism between quasicompact separated schemes, then $H^i(\mathscr{F}) \cong H^i(\pi_* \mathscr{F})$.

(4) Let C be an irreducible projective curve and $\tilde{C}$ its normalization. Show that the arithmetic genus of $\tilde{C}$ is less than or equal to the arithmetic genus of C. When does equality occur? Hint: Think of the naturall map $\mathcal{O}_C \to f_* \mathcal{O}_{\tilde{C}}$.

(5) Let S be a smooth surface and $C \subset S$ an integral curve with $\pi : \tilde{C} \to C$ the normalization. Show that the arithmetic genus of C is equal to $\deg(\pi^*(K_S + \mathcal{O}_S(C)))$. Hint: Use the adjunction formula for the dualizing sheaf of C.

(6) Show that a curve C of genus at least three cannot have maps to $\mathbb{P}^1$ of degrees both two and three. Hint: Consider the natural map $C \to \mathbb{P}^1 \times \mathbb{P}^1$. What is the arithmetic genus of the image?

(7) Show that there are non-hyperelliptic curves of any genus at least three.

(8) Show that a smooth plane curve of degree at least four is not hyperelliptic.