

Algebraic Geometry II
Homework 4
Due Friday, February 13

- (1) Recall that a closed immersion is an affine morphism $f : X \rightarrow Y$ such that the natural map of sheaves $\mathcal{O}_Y \rightarrow f_*\mathcal{O}_X$ is surjective. Show that we can replace the affine condition by the requirement that f is a homeomorphism onto a closed subset.
- (2) Show that if $i : X \rightarrow Y$ is a monomorphism, then $X \times_Y X \cong X$.
- (3) Show that the following classes of morphisms are closed under composition.
 - (a) Closed immersions.
 - (b) Separated morphisms.
 - (c) Proper morphisms.
- (4) Show that the following classes of morphisms are preserved under base change.
 - (a) Finite morphisms.
 - (b) Separated morphisms.
 - (c) Proper morphisms.
- (5) Show that a map from a separated k -scheme X to another k -scheme Y is separated.
- (6) Show that a map from a proper k -scheme X to another k -scheme Y is proper.
- (7) Show that the product of two irreducible varieties over an algebraically closed field is again an irreducible variety.
- (8) Show that a product of projective varieties is a projective variety.
- (9) Show directly that the affine line with a doubled origin is not separated, by showing its diagonal is not closed.
- (10) Give an example of a surjective morphism between two proper varieties such that the dimension of the fibers varies.