Algebraic Geometry II
Homework 7
Due Wednesday, March 18

(1) Show that the blowup of a point of $\mathbb{P}^2$ is isomorphic to $\mathbb{P}(\mathcal{O}_{\mathbb{P}^1} \oplus \mathcal{O}_{\mathbb{P}^1}(1))$.

(2) Let X be a smooth projective connected variety of dimension at least two. What is the relationship between the Picard group of X (the group of invertible sheaves on X) and the Picard group of the blowup of X at a point?

(3) Let f be a homogeneous polynomial in n variables. What is the exceptional divisor of the blow-up of $V(f) \subset \mathbb{A}^n$ at the origin?

(4) Let X be a finite type k-scheme and $p \in X$ a k-point. Show that the fiber is $\Omega_{X/k}$ at p is canonically isomorphic to the cotangent space of X at p.

(5) Show that if $Z \subset X$ is a smooth subvariety of a smooth variety, the exceptional divisor of $\mathrm{Bl}_Z X$ is the projectivization of the normal bundle of Z.