Algebraic Geometry II
Homework 8
Due Friday, April 3

(1) Let W be an affine space for a finite-dimensional vector space V, thought of as a scheme. Show that the cotangent space at any point of W is canonically isomorphic to the cotangent space at any other point.

(2) Let V and W be as above. Show that the cotangent space at any point of W is canonically isomorphic to V^*. Deduce that $TW \cong W \times V$.

(3) Fix a vector space V and a subspace $U \subset V$. Show that the space of splittings of the short exact sequence

$$0 \to U \to V \to U/V \to 0$$

is naturally an affine space for $Hom(U/V, U)$.

(4) Consider the Grassmannian $G(n, k)$ of k-dimensional quotients of an n-dimensional vector space V. Fix a k-dimensional subspace $U \subset V$. Show that the space of k-dimensional quotients of V with kernel disjoint from U can be identified with the space of splittings of the short exact sequence

$$0 \to U \to V \to U/V \to 0.$$

(5) Consider the Grassmannian $G(n, k)$ of k-dimensional quotients of an n-dimensional vector space V. Fix a k-dimensional subspace $U \subset V$. Consider the open subset of k-dimensional quotients of V with kernel disjoint from U. Show that we can identify the restriction of the tautological subbundle S with U/V and the restriction of the tautological quotient bundle Q with U.

(6) Show that the tangent bundle of the Grassmannian is isomorphic to $S^* \otimes Q$.

(7) Show that on $\mathbb{P}^n$ there is a short exact sequence

$$0 \to \mathcal{O} \to \mathcal{O}(1)^{n+1} \to T\mathbb{P}^n \to 0.$$