Algebraic Geometry II
Homework 9
Due Friday, April 10

(1) Calculate the cohomology of $T\mathbb{P}^n(d)$ and $T^*\mathbb{P}^n(d)$. Hint: use the exact sequence from the previous homework.

(2) Show that if A is a locally free sheaf, $A \otimes \cdot$ and $\mathcal{H}om(A, \cdot)$ are exact.

(3) Calculate the cohomology of the structure sheaf of $\mathbb{A}^2 \setminus 0$.

(4) On a projective scheme over a field, define $\chi(E) = \sum(-1)^i \dim H^i(E)$, which will be finite. Show that given a short exact sequence

$$0 \to A \to B \to C \to 0$$

we have $\chi(B) = \chi(A) + \chi(C)$.

(5) Let X be a hypersurface in $\mathbb{P}^n$ of degree d. Calculate $\chi(\mathcal{O}_X)$.

(6) What is the Euler characteristic of a connected projective curve of genus g?

(7) Show that if Z is a complete intersection of two effective Cartier divisors D_1 and D_2 on X, we have an exact sequence

$$0 \to \mathcal{O}_X(-D_1 - D_2) \to \mathcal{O}_X(-D_1) \oplus \mathcal{O}_X(-D_2) \to \mathcal{O}_X \to \mathcal{O}_Z \to 0.$$

(8) What is the genus of a smooth curve which is the intersection of a quadric and a cubic in $\mathbb{P}^3$?