Algebraic Geometry II
Homework 4
Due Friday, February 13

(1) Recall that a closed immersion is an affine morphism $f : X \to Y$ such that the natural map of sheaves $\mathcal{O}_Y \to f_*\mathcal{O}_X$ is surjective. Show that we can replace the affine condition by the requirement that f is a homeomorphism onto a closed subset.

(2) Show that if $i : X \to Y$ is a monomorphism, then $X \times_Y X \cong X$.

(3) Show that the following classes of morphisms are closed under composition.
   (a) Closed immersions.
   (b) Separated morphisms.
   (c) Proper morphisms.

(4) Show that the following classes of morphisms are preserved under base change.
   (a) Finite morphisms.
   (b) Separated morphisms.
   (c) Proper morphisms.

(5) Show that a map from a separated k-scheme X to another k-scheme Y is separated.

(6) Show that a map from a proper k-scheme X to another k-scheme Y is proper.

(7) Show that the product of two irreducible varieties over an algebraically closed field is again an irreducible variety.

(8) Show that a product of projective varieties is a projective variety.

(9) Show directly that the affine line with a doubled origin is not separated, by showing its diagonal is not closed.

(10) Give an example of a surjective morphism between two proper varieties such that the dimension of the fibers varies.